Towards a general solution of the pressure drop characteristic of meandered channel flow

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Summary

A model of meandering channel flow was implemented in ANSYS CFX® 11 update1. The SST turbulence model with automatic wall functions was applied with approximately 3*E+5 nodes and 2.5*E+5 elements. The residual target was 5*E-5 and one typical simulation lasted 4*E+3 seconds on a Pentium 4 CPU with 3 GHz and 2 GB Ram. Double precision was always used. The model is scalable with respect to width B, length L and depth T of one rectangular channel, the entire model consists of 6 channels. The wall length L1 where meandering takes place is less then L. The ratio L1/L is a characteristic parameter of minor interest for this investigation. The channel flow is completely closed, with one inlet and one outlet, where the pressure drop was looked at. A parameter study was done with the model in the parameter range L, T, B [0.4…1.6, 0.005…0.015, 0.1…0.2] [m] for one constant flow rate Vp of 1.5 [m3/h]. The calculated pressure drop, compared to measurement results, showed a good agreement regarding them as minimum values. Finally non-dimensional relationships between pressure drop and geometry parameters were extracted from simulation results.

Keywords

pressure drop, meandered channel flow, ANSYS/CFX, parameter study

1. Symbols

Symbol	Unit	Description
B0	m	inward width of the channel
B*	m	constant width
B1	m	thickness of wall
L	m	length of the channel
L0	m	specified minimum length
L1	m	length of inward separation
LC	m	length of inward sheath in curve
Nx	~	number of meandering segments forming the channel
T	m	depth of the channel
λ	~	pipe resistance value
$\lambda*$	~	reformulated pipe resistance value by equation (7)
Δp	Pa	pressure drop of the total channel
v	m/s	velocity in the main channel
ρ	kg/m^3	density of the fluid – here water
$\dot{V}$	m^3/s	volume flow of the fluid
d	m	diameter, characteristic length
k		constant
k*		constant
A	~	polynomial coefficient
n	~	potential coefficient
T&B0	~	index number of the different constant T & B combinations for experimental set up
pf		calculated polynomial function (X/)
WM_Nr		Index number of different water coatings according to different settings for B0,T,L

Index	Description
s	for one segment, i.e. one straight piece of channel
g	for the whole channel with Nx segments
h	hydraulic
d	detailed
i	integer index number of a specified range
k	related to coefficients
b	related to B0 variations

2. Introduction

The pressure drop characteristic of a meandering flow inside a water coating used for cooling of electrical machines is measured as a standard for quality control. To get evidence what the predicted values would be a CFX model was implemented to compare experimental findings and numerical calculations. In the following investigation it was tried to find analytical solutions in a generalized form from the numerical results from volume related parameters, i.e the length, the width and the depth of the channel.

3. Model

The channel model implemented in CFX can be characterized by Fig. 1. There is an input flow with equally distributed streamlines. In the CFX model investigated six channels where modeled. The pressure drop solutions for more channels are supposed to increase linearly with the number of channels.

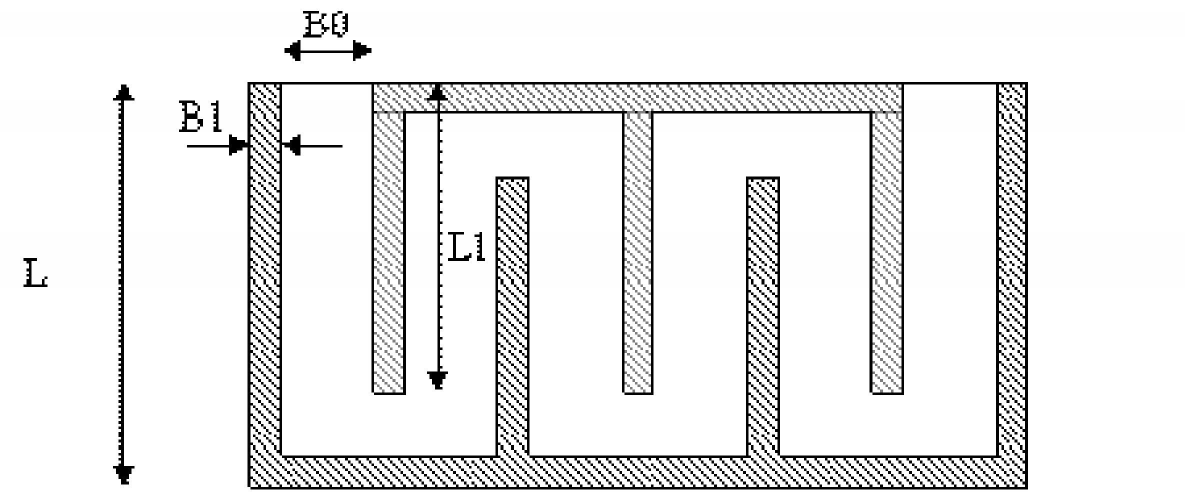

Fig. 1 Schematic CFX flow model of meandered channel flow. The depth of the channel is T

4. The range of the solution matrix

A generalized model is supposed to be described with a matrix of parameters of L, T and B0. These parameters are limited by values of built water coatings. The limits of the experimental coatings corresponding to the numerical investigations are in the size range shown in Tab. 1.

Tab. 1 Range of the solution matrix and multiplication factor of the size range

	L	T	B0
Min	400	5	100
Max	1600	15	200
x-factor	4	3	2

4. Generalization of the parameters for channel width B0 and length L lamda =f(B0,L) for Experimental Set up

4.1 Introduction

For the experimental set up the pressure drop was measured. The volume flow $\dot{V}$ was constant for all measurements so it was no parameter for the following investigations. The pipe resistance value can be written in general according to e.g. [1] with (1):

$$\lambda = \frac{\Delta p}{\frac{\rho}{2} \cdot v^2} \cdot \frac{d_h}{L_g} \qquad (1)$$

For the channel flow (1) can be modified with respect to the volume flow $\dot{V}$ and a simplified hydraulic diameter $d_h = 2 \cdot T$:

$$\lambda = \frac{\Delta p}{\frac{\rho}{2} \cdot \left(\frac{\dot{V}}{B0 \cdot T} \right)^2} \cdot \frac{2 \cdot T}{Nx \cdot L} \qquad (2)$$

a simplification gives:

$$\lambda = \frac{4 \cdot \Delta p}{\rho \cdot \dot{V}^2} \cdot \frac{T^3 \cdot B0^2}{Nx \cdot L} \qquad (3)$$

The detailed solution for the hydraulic diameter d_h is:

$$d_h = \frac{2 \cdot T}{1 + \frac{T}{B0}} \tag{4}$$

Equation (4) inserted in (3) yields the detailed solution for the dimensionless pipe resistance value:

$$\lambda_d = \frac{4 \cdot \Delta p}{\rho \cdot \dot{V}^2} \cdot \frac{T^3 \cdot B0^2}{Nx \cdot L} \cdot \frac{1}{1 + \frac{T}{B0}} \tag{5}$$

The relation to the approximation function for lamda is then:

$$\lambda_d = \frac{\lambda}{1 + \frac{T}{B0}} \tag{6}$$

It can be concluded that the simplified solution can be used für B0>>T, which ist the case for all experimental setups and hence $\lambda_d \sim \lambda$.

5.2 Generalization for lamda=f(L/L0)

For the experimental set up a TB0 matrix can be formulated for 23 combinations of T and B0, where only parameters for L are varying.

Tab. 2 T&B0 matrix depicting the experimental range of T and B0 values with corresponding numbers (T&B0_i, i=1..23) used in Fig. 3

T\B0	109	119	123	138	144	146	147	153	155	160	164	165	170	174	184	186	193	194	208	219
5	4		1																	
8							12		11	10	9	8	6	5	3	2				
10						14													7	
12		23		22	20	19		18			16						13			
15														21				17		15

Those combinations can be depicted with respect to T and B0 in Fig. 2.

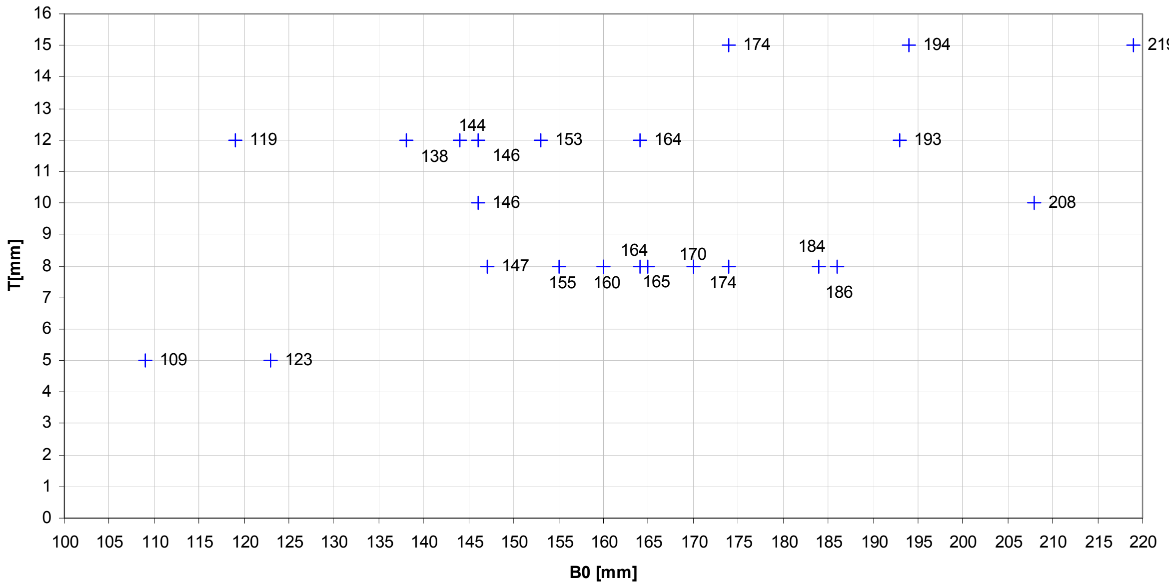

Fig. 2 T&B0 Matrix for experimental set up

The solution for λ according to (3) and a variety of 23 combinations of B0 & T is shown in Fig. 3.

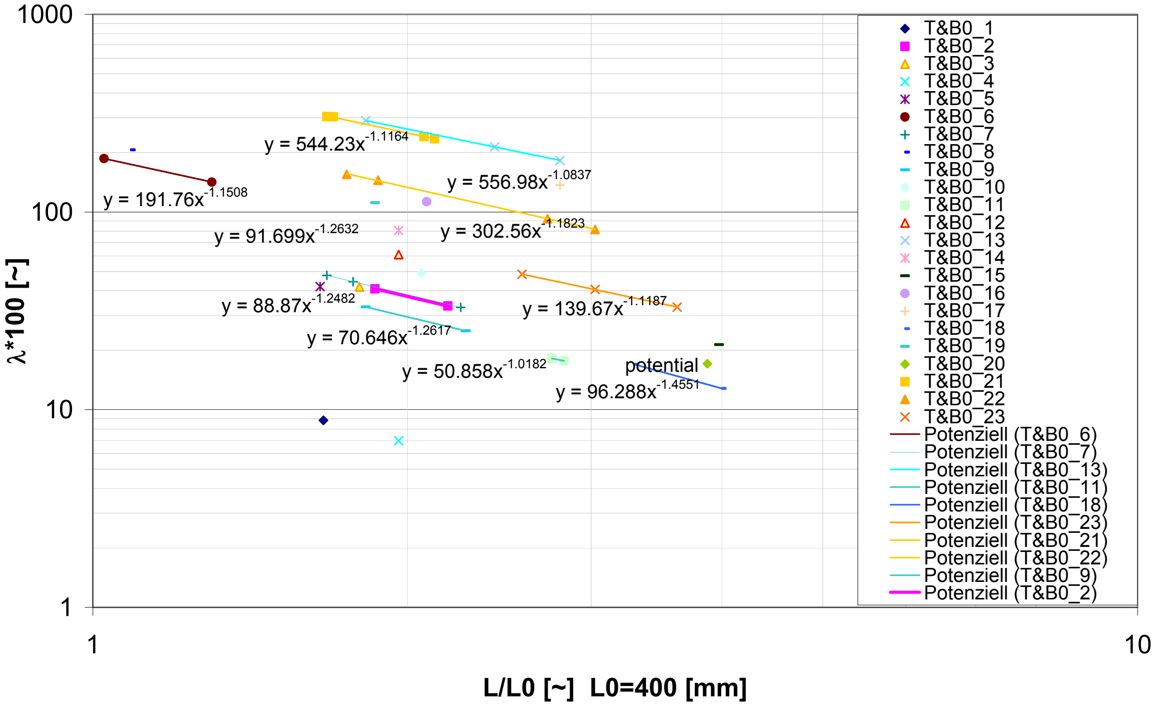

Fig. 3 Numerical solution for the pipe resistance value for the experimental set up

The polynomial fittings are corresponding to constant T and B0 combinations with different lengths. The coefficients for the potential function

$$\lambda_d = A \cdot \left(\frac{L}{L0} \right)^n \qquad (7)$$

are shown in Tab. 3.

Tab. 3 polynomial fittings for several type series i.e. water coatings with different lengths for constant T&B0; the column pf is used for polynomial fittings in Tab. 5 and Fig. 7.

T [mm]	B0 [mm]	T&B0 [~]	A [~]	n [~]	pf [~]
8	155	11	50.858	-1.0182	
8	164	9	70.646	-1.2617	X
8	170	6	191.76	-1.1508	
8	186	2	88.87	-1.2482	X
10	208	7	91.699	-1.2632	X
12	119	23	139.67	-1.1187	X
12	138	22	302.56	-1.1823	X
12	153	18	96.288	-1.4551	
12	193	13	556.98	-1.0837	X
15	174	21	544.23	-1.1164	X

5.3 Generalization for lamda=f(B*/B0)

To get samples for λ with constant L/L0 and different B0 (7) was used to calculate the λ's for the different type series represented by the potential functions for a range of B0's. As can be seen from

Fig. 4 for T=12 the λ's vs. 1/B0 lie on a straight line. For the least squares line fit those for lower L/L0 are increasingly better (Tab. 4).

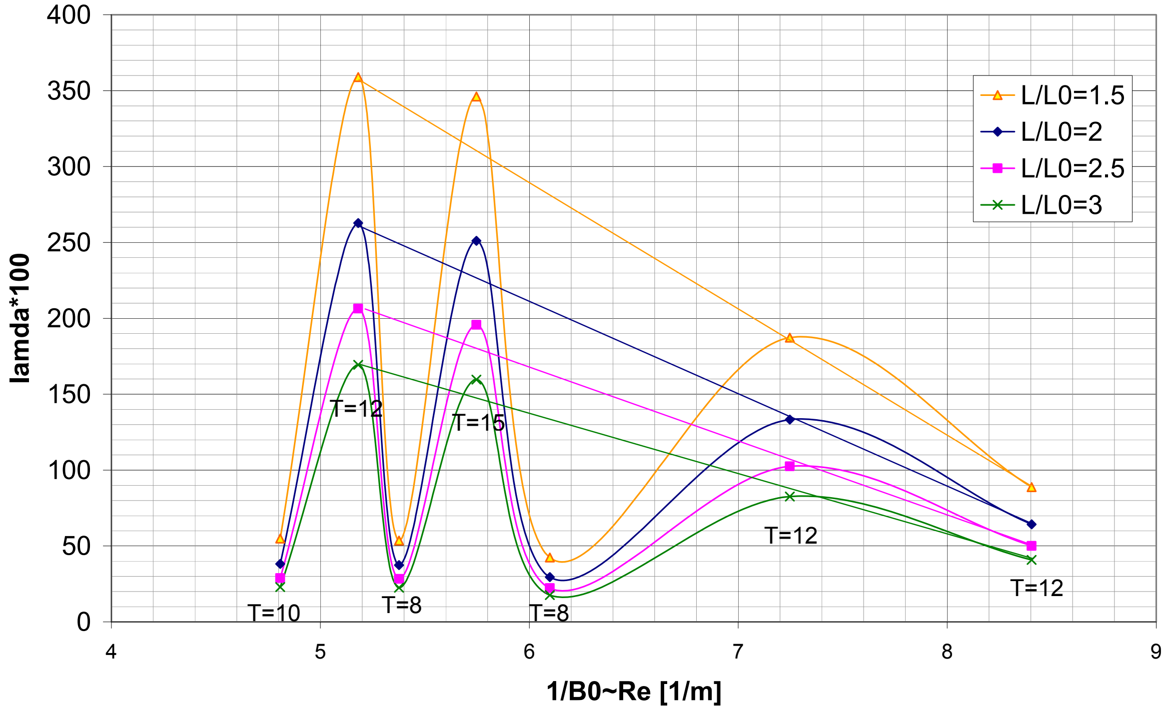

Fig. 4 Calculated λ's with (7) and coefficients from Tab. 3 for four values of L/L0; Equal T's yield "nearly" a straight line ; T in [mm]

It can be concluded that for one type series λ is inverse proportional to the width of the channel. So what is the mathematical relation?

A straight line can be defined according to:

$$\lambda = k \cdot \left(\frac{B^*}{B0} \right) + d \qquad (8)$$

In Fig. 5 (8) is shown with data from Tab. 3 for data with T=12 only.

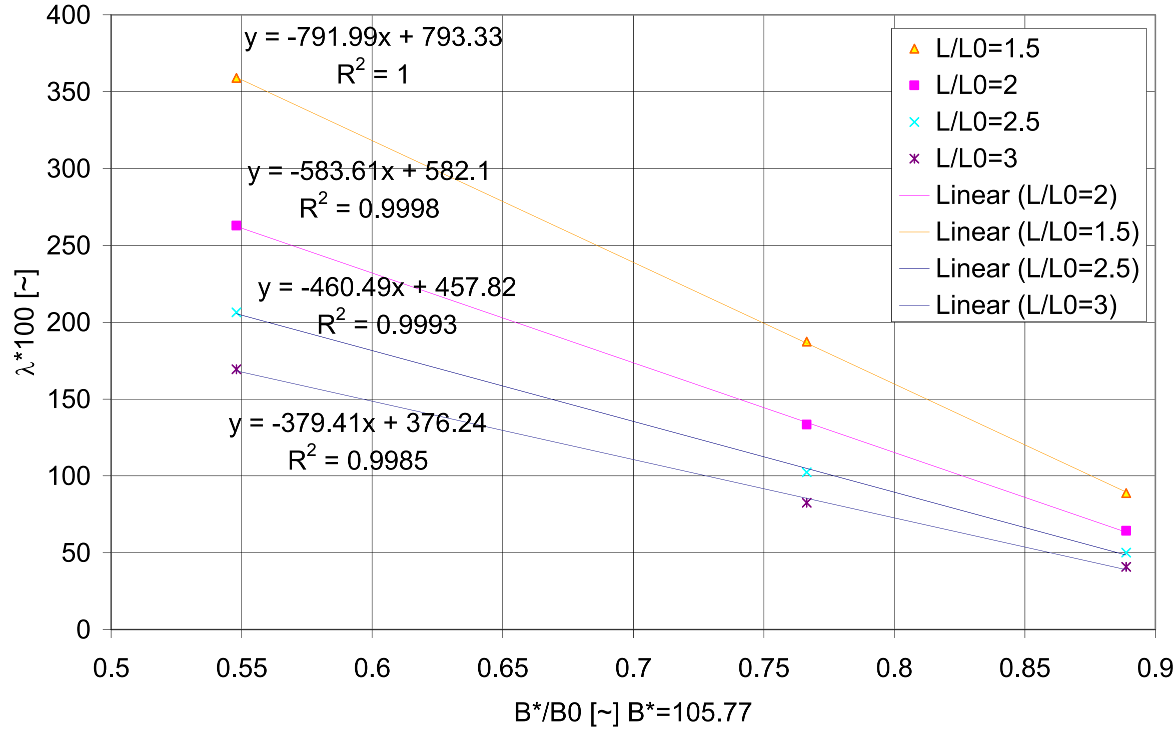

Fig. 5 Calculated λ's with (7) and coefficients from Tab. 3 as a Function of B*/b0 b*=105.77 [mm]

B* was determined by maximizing all the regression coefficients r^2 with B*=105.77. The coefficients are shown in Tab. 4.

Tab. 4 Linear Coefficients of straight lines according to (8) and the data of Fig. 4.

L/L0	k	d	r^2
1.5	-791.99	793.33	1
2	-583.61	582.1	.9998
2.5	-460.49	457.82	.9993
3	-379.41	376.24	.9985

A potential fit with the d's gives:

$$d_i = A_k \cdot \left(\frac{L}{L0}\right)^{n_k} = 1227.4 \cdot \left(\frac{L}{L0}\right)^{-1.0763} \qquad (9)$$

with A_k=1227.4 and n_k=-1.0763 for T=0.012.

5.4 Generalization for lamda=f(L/L0,B*/B0)

When we suppose now that both generalizations can be united, assuming that the k's and d's are approximately one constant k_b (d=-k=k_b) giving (10) from (8).

$$\lambda = k_b \cdot \left(1 - \left(\frac{B^*}{B0}\right)\right) \qquad (10)$$

We can conclude from (9) with k_b=d_i that λ can be expressed as a function of L/L0 and B*/B:

$$\lambda = A_k \cdot \left(\frac{L}{L0}\right)^{n_k} \cdot \left(1 - \left(\frac{B^*}{B0}\right)\right) \qquad (11)$$

In (11) B* is the uniting coefficient for the different k_b's. It is not necessarily the same for different combinations of B0's.

When we now reformulate λ by dividing through B*/B0 we yield by definition

$$\lambda^* = \frac{\lambda}{\left(1-\left(\frac{B^*}{B0}\right)\right)} \tag{12}$$

and with (11)

$$\lambda^* = A_k \cdot \left(\frac{L}{L0}\right)^{n_k} \tag{13}$$

That means that λ^* is only a potential function of L/L0 with T=const for a defined type series (L/L0 is varying). The solution for T=0.012 is given in Fig. 6.

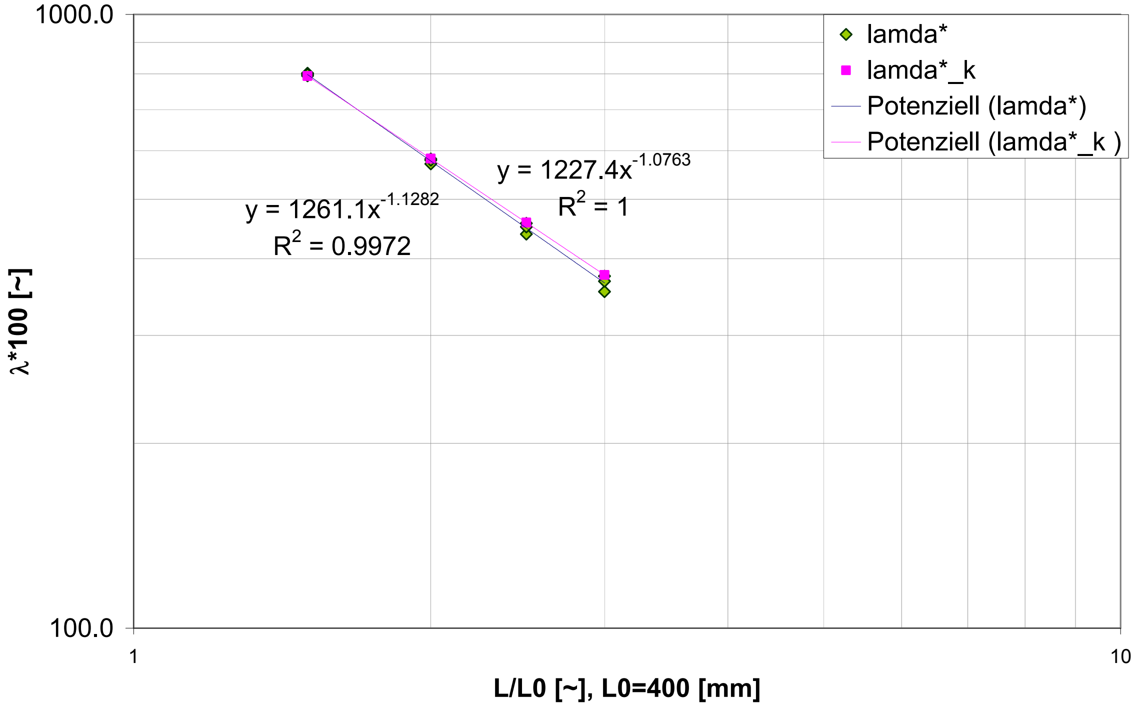

Fig. 6 lamda*_k according to (9) and lamda* according to a Potential Fit for the data in Tab. 3 for T&B0_i (i=13,22,23)

For different T and all the data in Tab. 4 with pf=x the potential functions are depicted in Fig. 7. The parameters n_k and A_k are then different.

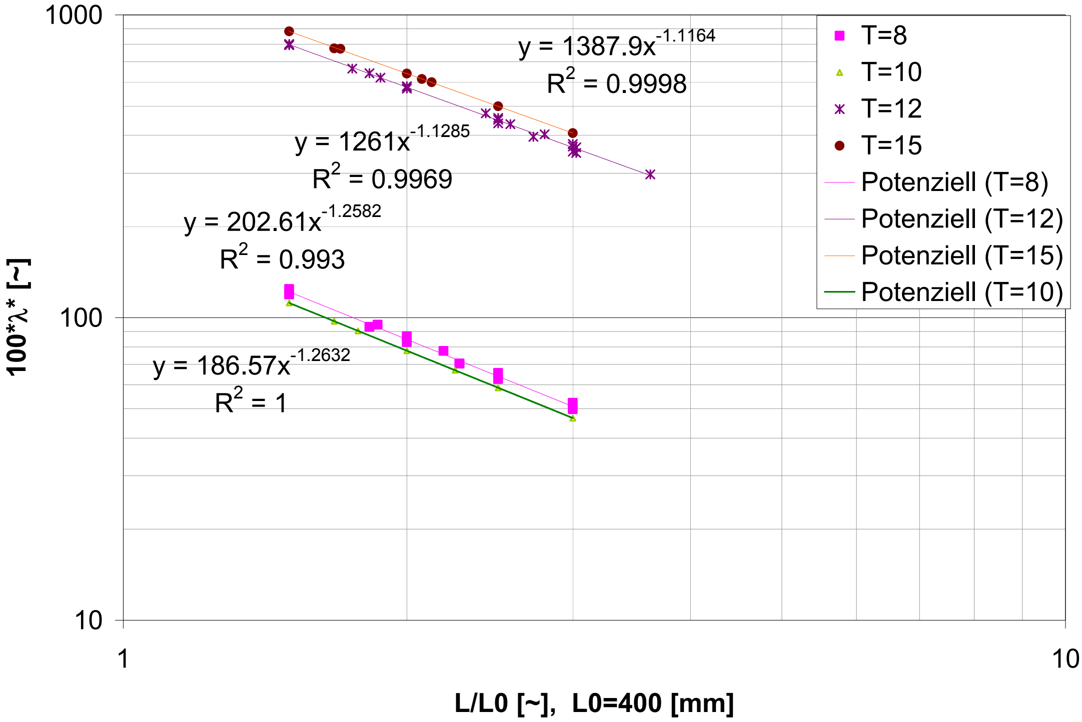

Fig. 7 lamda* as a function of L/L0 for all values in Tab. 3 with pf=x

Tab. 5 Data for Fig. 7

T	**B0**	**A_k**	**n_k**	**r^2**
[mm]	**[mm]**	**[~]**	**[~]**	**[~]**
8	164/186	206.61	-1.2582	.993
10	208	186.57	-1.2632	1
12	119/138/193	1261	-1.1285	.9969
15	174	1387.9	-1.1164	.9998

For every WM_Nr the data are depicted in the following diagram:

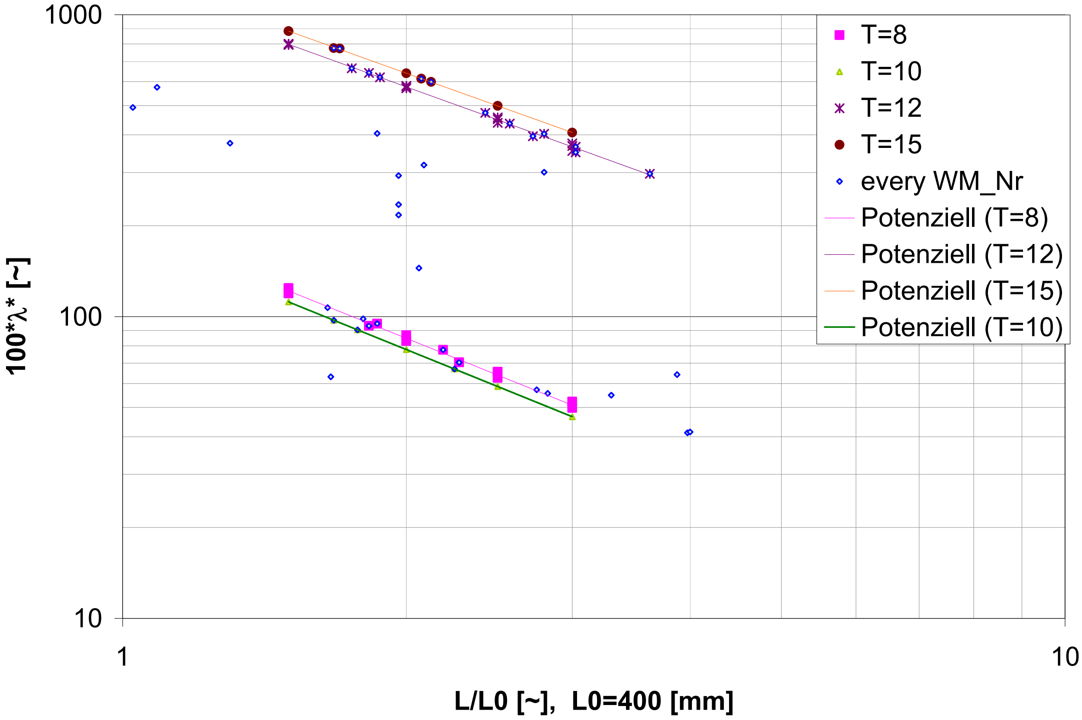

Fig. 8 lamda* as a function of L/L0 for all values in Tab. 3 with pf=x and for every WM_Nr

So it seems that there is a possible generalization with respect to B*/B0 for a certain T=const. This generalization does not yet work when trying to generalize for T. Anyhow there is a potential function for each set of constant T&B0, as can be followed from Fig. 3.

For constant L/L0 there seems to be a potential function for constant B0 however, which is shown in the following diagram for WM_Nr=8:

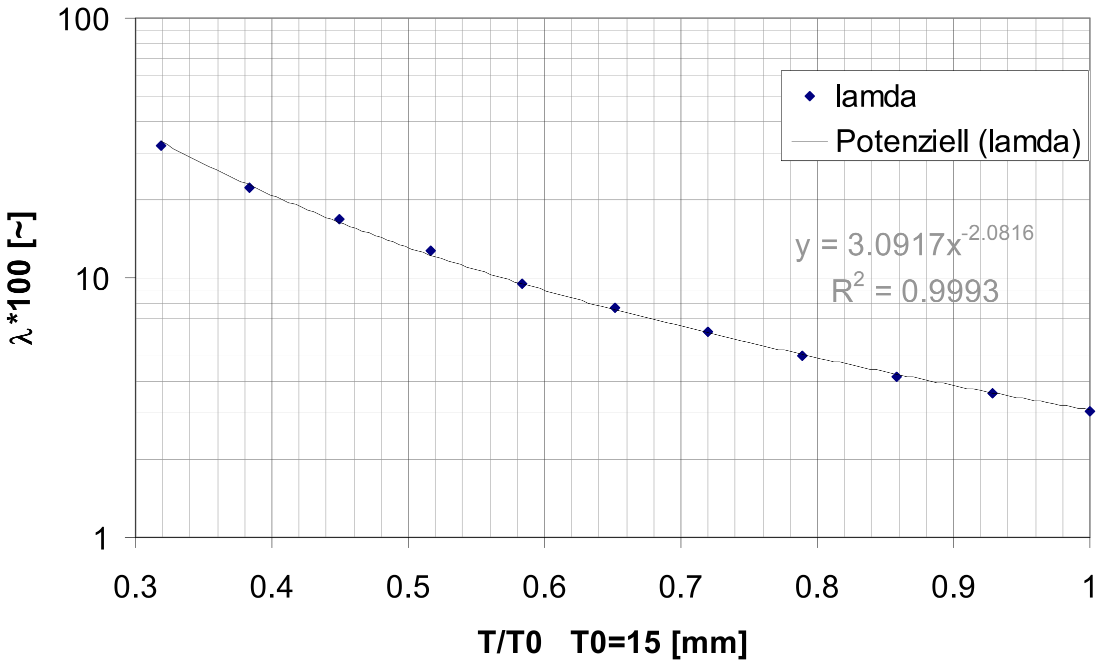

Fig. 9

6. Conclusion

Concerning the generalization with respect to L/L0 and B*/B0 it can be concluded as a summary:

- one set with T&B0 (i.e. one type series) can be fitted with a potential function. *With increasing length* L/L0 the pipe resistance λ *is decreasing*.
- at least for a "specific set" of type series a simultaneous fit of lamda for length L/L0 and width of the channel B*/B0 is possible indicating that a) there is a limit for a minimum length B* with minimum pipe resistance (concluded from (11)) and that b) the pipe resistance λ is *increasing asymptotically with B0*, i.e. slower with greater widths. In the asymptotic case B0→∞ λ is independent of B*/B0 (compare with (11)).

There remain several questions:

- What is the parameter n_k and A_k for the potential function $\lambda*$ in general?
- For which conditions B* is the same and what is the role of B*?
- How can the influence of the parameter T be expressed?
- Is there a dependence of some kind of fractal volume defined by dimensions L,B0,T?
- Is there missing a geometry parameter in description of the general model?
- Is there a balance around the minimum width B*?

7. Acknowledgment

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8. References

[1] Zlokarnik M., Dimensional Analysis and Scale-up in Chemical Engineering, Springer Verlag, Berlin, 1991

[2] Gotter G., Erwärmung und Kühlung elektrischer Maschinen, Springer Verlag, Berlin, 1954